THE
LIGHT CLOCK
WAS AN
ESCHER PICTURE

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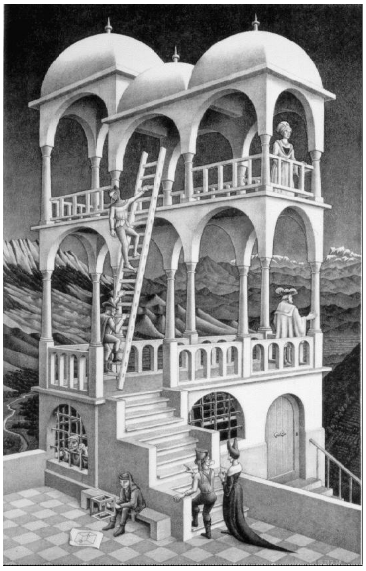
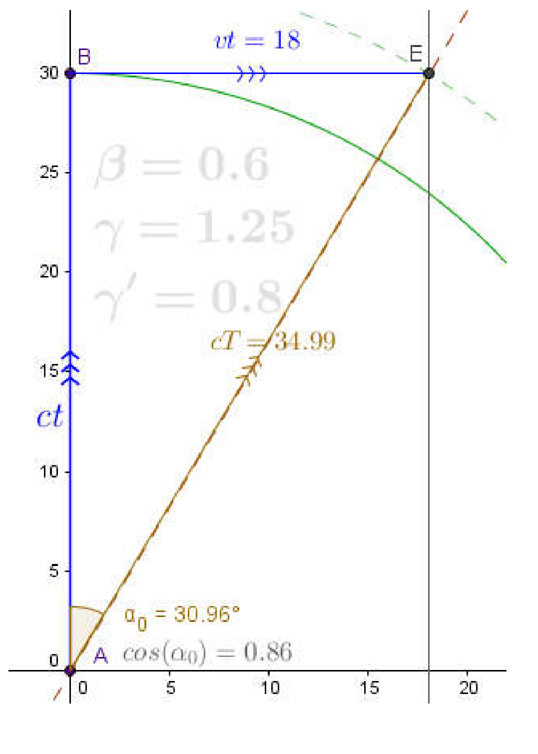


ABSTRACT

The moving light clock is the most prominent example in relativity to introduce the Lorentz factor. According to the constant speed of light the diagonal paths of light are assumed to exceed the vertical paths in terms of distance and in terms of time as well, but they do not, the derivation of the Lorentz factor suggests. The light clock is applied two reference sizes for velocities. Both denote the motion along the same direction but use different reference magnitudes. The formulas commonly used omit the vector notation and introduced a fatal error in relativity.

THE LIGHT CLOCK

The light clock[1] installs a virtual object bouncing vertically between two mirrors at the speed of light. An observer moving with velocity $v = dx/dt$ records the distance traversed by the light. His readings exceed the strict vertical distance between the mirrors. Following the principle of invariance of light speed the travel times are expected to increase, *but they don't have to*, as the derivation of the Lorentz factor seems to show. The observer of the light clock records the following shape. The variables t and T denote distinct references of time.



$$c = 1$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma' = \sqrt{1 - \frac{v^2}{c^2}}$$

$$(ct)^2 + (vt)^2 = (cT)^2$$

$$c^2t^2 + v^2t^2 = c^2T^2$$

$$\frac{T^2}{t^2} = \frac{c^2 + v^2}{c^2}$$

$$T = \sqrt{1 + \frac{c^2}{v^2}}\, t$$

The term $a^2 + b^2 = c^2$ implies a shape having a right angle between lines indicated by a and b. For the light clock to operate the velocity v has to *change reference* from t to T. To switch reference, e.g. to use inches instead of meters, implies to preserve the *meaning* of $\overrightarrow{vt}$ such that the *shapes* referring to it *do not* change.

To keep the magnitude of $\overrightarrow{vt}$ with respect to the direction indicated it is necessary to convert v to V such that

$$\overrightarrow{VT} = \overrightarrow{vt}.$$

But we have to consider that

- the direction of cT is not perpendicular to the x-axis and
- the magnitude of T is exceeding the magnitude of t.

From this we must conclude that we don't have some invariant to give a *plausible conversion rule* for v into V.

The ratio of T and t is given by the shape as:

$$\frac{T}{t} = \frac{cT}{ct}$$

The transfer of v from t to T must keep the whole area invariant that is affected by light during motion, otherwise both models would not be equivalent:

$$v\ ct = V\ cT.$$

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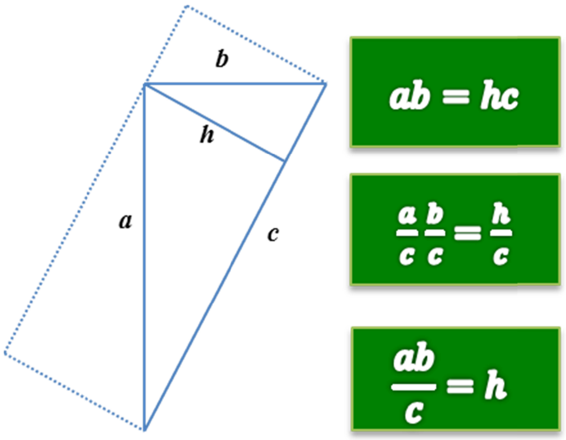
<td>
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<td>The area $A = ab/2$ is equal to the area given by $B = hc/2$ as the doubled areas $2A = 2B$ are equal.

If c is set to 1, there is some simplification form.</td>
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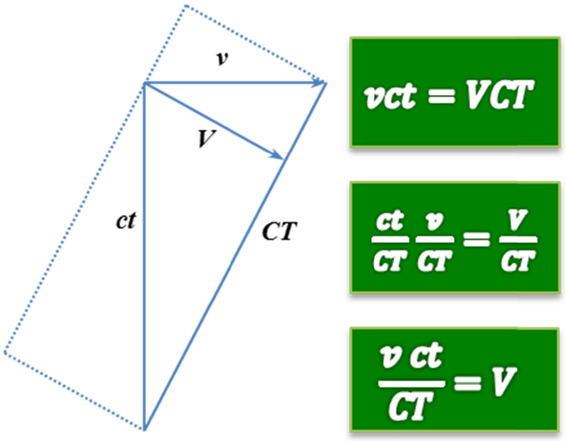
<td>
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<td>The value of t may be provided.

As ct and vt were given as interdependent the vector $\overrightarrow{vt}$ may not be converted as stand alone.

When switching v from t to T, the vector $\overrightarrow{VT}$ may not be derived from $\overrightarrow{vt}$ only.

If the area $$v\ ct = V\ cT$$ is kept invariant the shape will not change.</td>
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By applying the invariant

$$v\ ct = V\ cT$$

we deduce the conversion rule

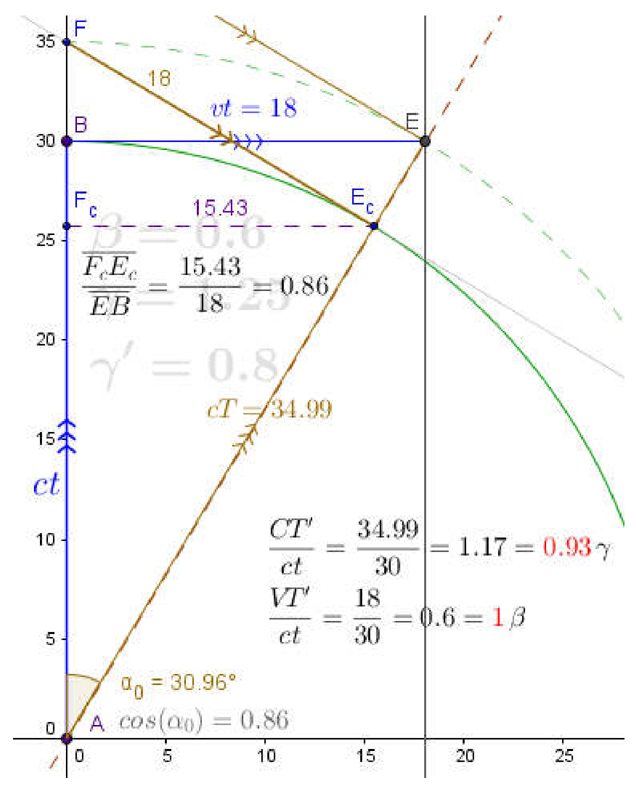
$$V = v\frac{ct}{cT}$$

and exchange vt by VT for the primary term, hence we get:

$$(ct)^2 + (VT)^2 = (cT)^2$$

$$(ct)^2 + \left(\left(v\frac{ct}{cT}\right)T\right)^2 = (cT)^2$$

$$(ct)^2 + (vt)^2 = (cT)^2.$$



$$c = 1$$

$$\beta = \frac{v}{c}$$

$$(ct)^2 + (VT)^2 = (cT)^2$$

$$(ct)^2 + \left(\left(v\frac{ct}{cT}\right)T\right)^2 = (cT)^2$$

$$c^2t^2 + v^2\frac{t^2}{T^2}T^2 = c^2T^2$$

$$\frac{T^2}{t^2} = \frac{c^2 + v^2}{c^2}$$

$$T = \sqrt{1 + \frac{c^2}{v^2}}\, t$$

Note that

$$\beta = \frac{v}{c}$$

is indicating the speed of the light clock with respect to $v = dx/dt$. As v is **unchanged** the factors γ and γ' did not change. The shape did not change and the formula to describe the geometry of the light clock is still the same.

Conclusion

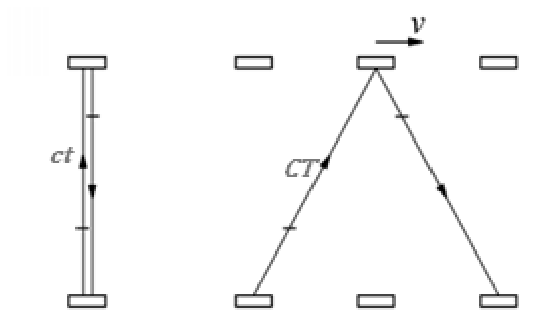
If we preserve the area $vct = VCT$ we preserve the geometrical shape of the light clock. If we preserve the shape there is preserved the tangens-ratio of T to t,

$$T = \sqrt{1 + \frac{c^2}{v^2}}\, t,$$

indicating $tan(\alpha) = v/c$ which does not imply constraints to velocities $v = dx/dt$ along the x-axis. There is no chance to produce some Lorentz-factor by light clocks.

The Light Clock as commonly taught in Relativity

The moving light clock is the most prominent example in relativity to introduce the Lorentz factor. It is used in textbooks, relativity courses and online courses[2] as well.



Examples[3]:

“All moving clocks are slowed by motion.”

“The light blips in both travel at the same speed relative to us.”

“This means that time vary for frames at different velocities with respect to each other.”[4]

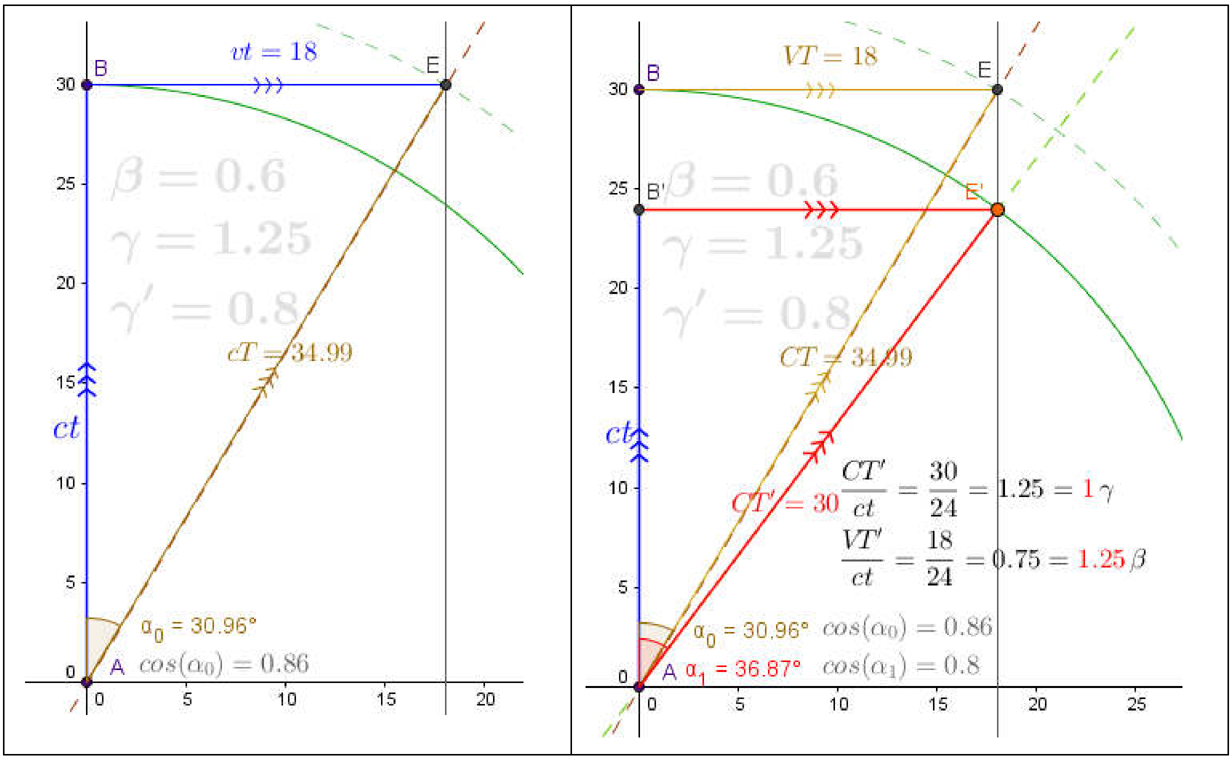
The light clock implies the unique ratio

$$\frac{CT}{ct} = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

for v towards the $+x$-direction.

- If ratio $\beta = v/c$ refers to vt/ct it gives indication that
  $v = dx/dt$ and
  $v\ ct$ gives the enveloping area of light during motion, by $(c \times v)\ dt$.
- If ratio $\beta = v/c$ refers to $vT/cT = VT/CT$ it gives indication that
  $v = dx/dT$ and
  $v\ CT$ gives the enveloping area, by $(c \times v)\ dT$.

In order to get clarity about the vectors and the relationships to their driving variables we use V in conjunction with T and v with t analogously.



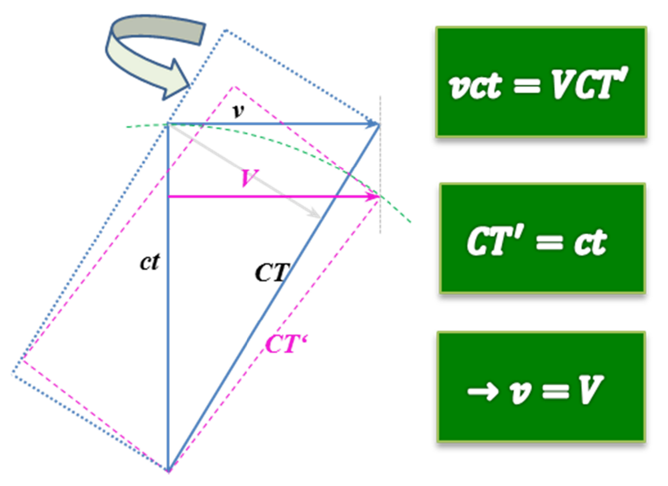
The 1:1 conversion of the β-factor, following $\overrightarrow{vt} = \overrightarrow{VT}$, does not preserve the shape.

- The switch to the reference magnitude of VT **increases the *effective speed* of v** by the factor γ.
- The area affected by motion is downsized by γ, the same factor the corresponding value of ct was reduced.

If we compare the area enclosed by $AB'E'$ with that of ABE the ratio $\overline{AB}/\overline{AB'}$ to acknowledge is enough to give the ratio. The segment $\overline{BE} = \overline{BE'}$, which represents the 1:1 conversion, is common to both.

> Despite this obvious discrepancy between both areas, the invariant, given as
>
> $$v\ ct = V\ CT'$$
>
> is fulfilled.

By geometrical construction the distance CT' may be proven to be equal to ct easily. Hence the equation above is fulfilled, if $v = V$. This identity is satisfied by geometrical construction and the 1:1 conversion as well.



VCT' will equal the area ***vct*** **if** we rotate ***V*** to become perpendicular to ***CT***.

When rotating ***V*** to become perpendicular to ***CT*** we will see that ***CT'*** will match ***ct***.

How to resolve this discrepancy?

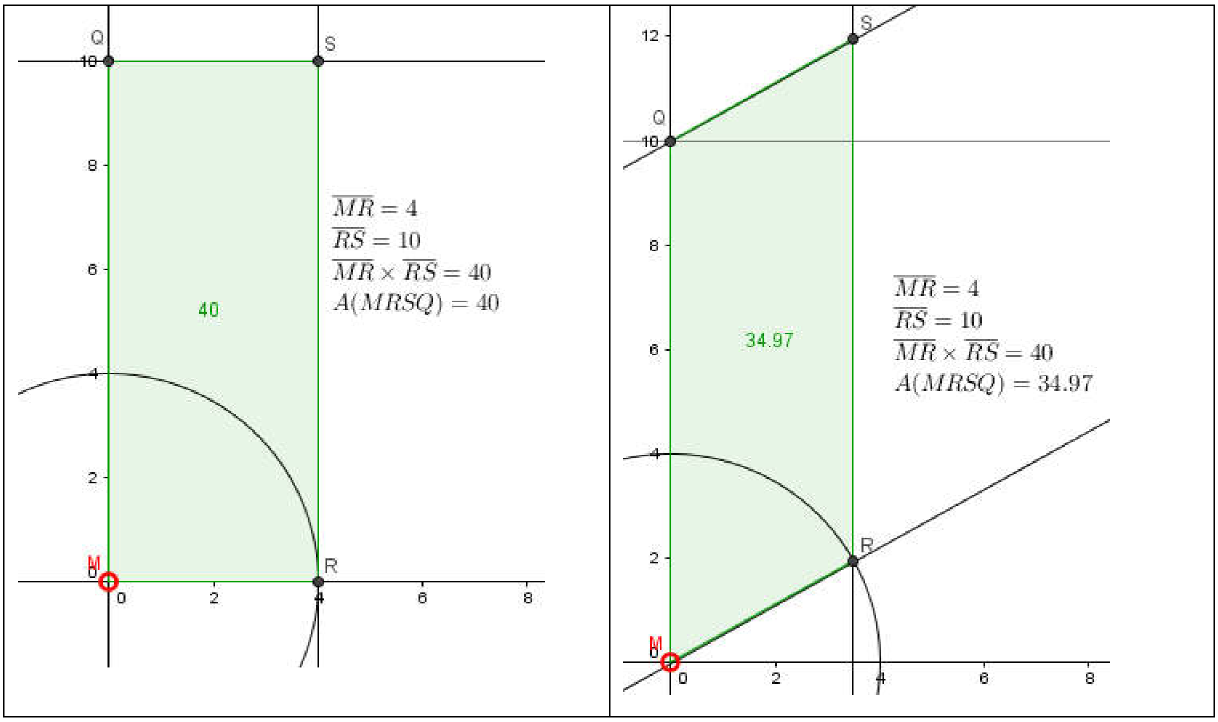
The invariant

$$v\ ct = V\ CT'$$

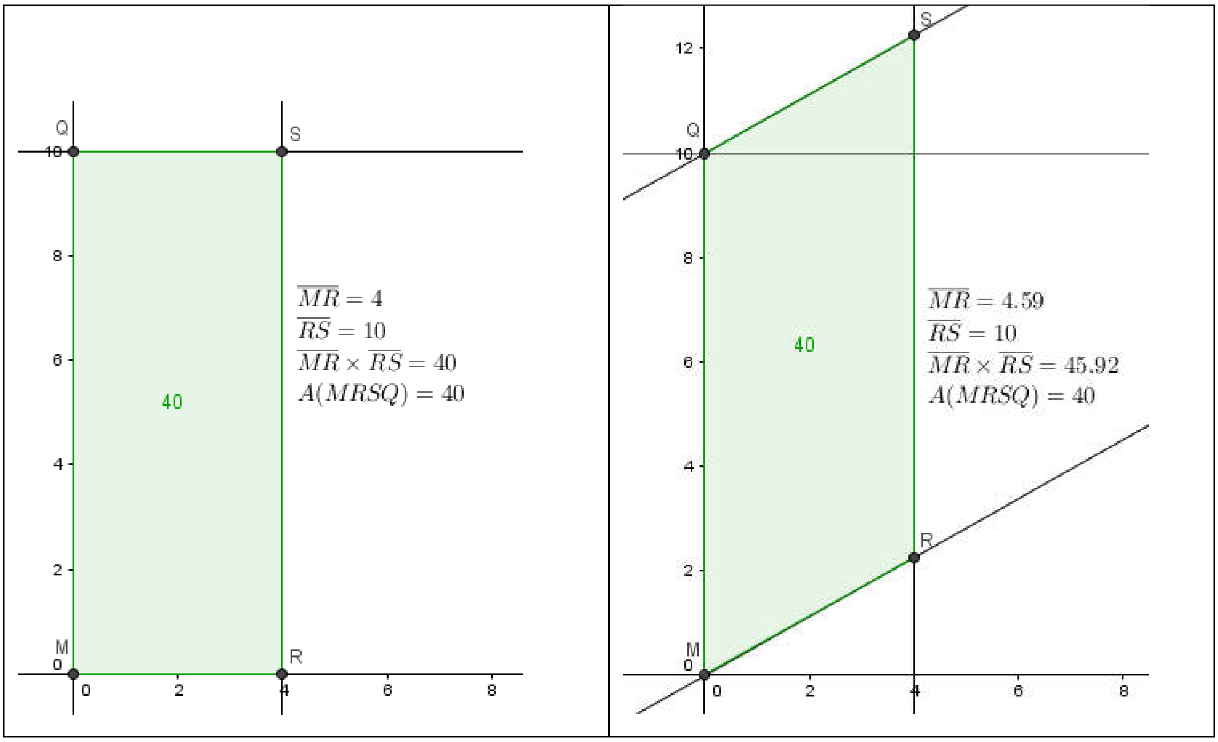
refers to the **product** of two numbers V and CT'. The product of two numbers N and n gives the n-fold sum of N. If we apply the v-fold to ct we will have the equivalent area $v \times ct$, if we apply the V-fold to CT' we would go astray. The result gives the V-fold of ct because V is perpendicular to ct and not perpendicular to CT'.

> Any products of two numbers that indicate the magnitude of **vectors** imply the corresponding vectors to be **perpendicular** to each other.

To illustrate this **serious mistake** built in the light clock as taught in common physics lectures the following examples may be useful.



If we change a parallelogram shape such that the width remains constant, we might suppose to use a simple product of two numbers to get the area. We may do so by short-cut only, because the transition from rectangles to parallelograms does not change the height and leaves the width untouched, which is to keep the product of length and height invariant.

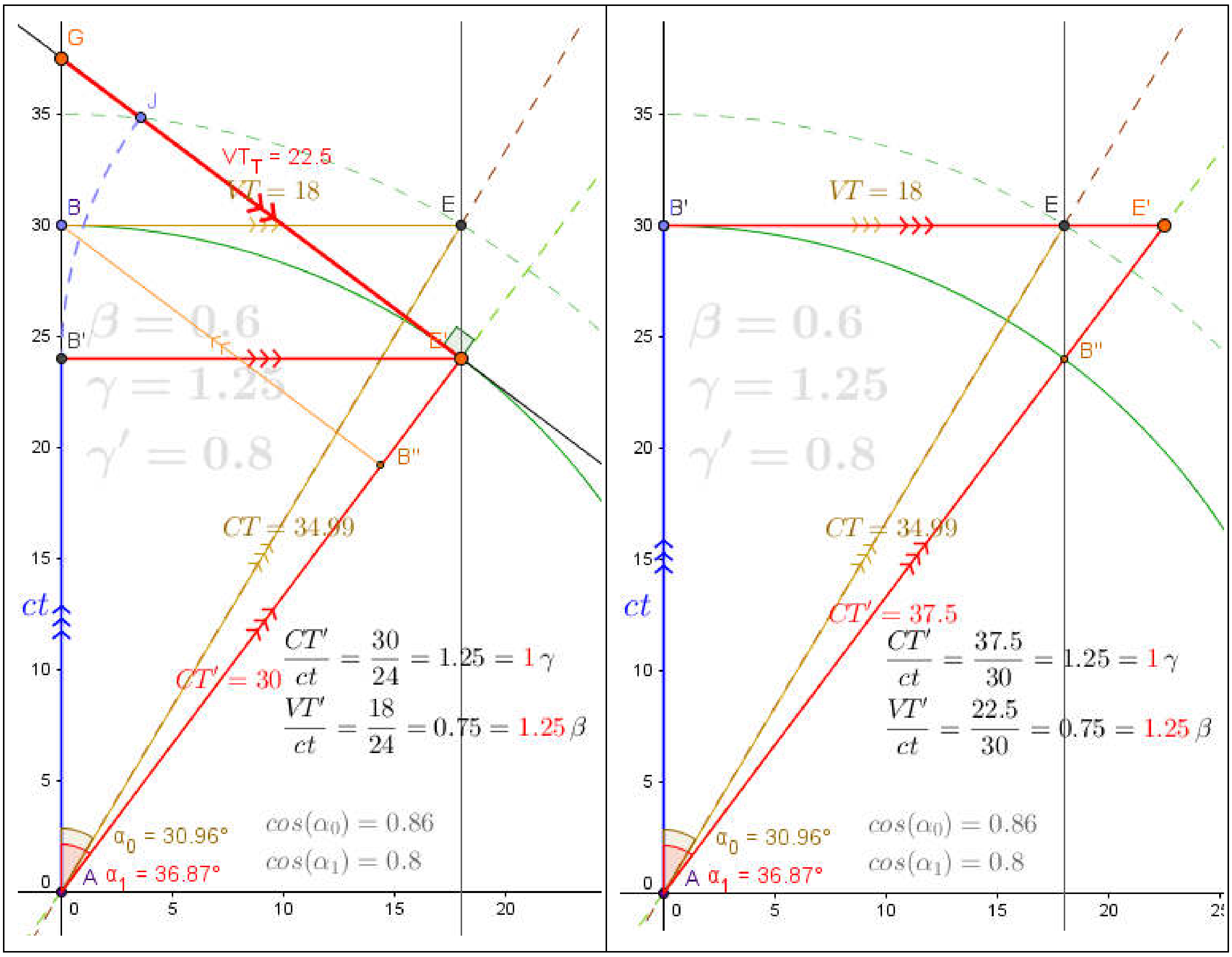


Correction

The light clock as common understood implies a speed-up of

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

The light clock delivers free energy as speed $\vec{v}$ gets a free boost of γ. This energy is free, if we first assume to have some speed v related to t and *then to* switch the same magnitude v to the newly created reference size $T = t\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.



The light clock, when used to derive the Lorentz factor, needs a patch. The indicated speed may refer to $v = dx/dt$ or to $V = dx/dT$. The angle α enclosed between the x-axis and ct is $\pi/2$. The angle α_1 between the x-axis and CT is $\sin(\alpha_1) = \frac{VT}{CT}$. Therefore, we have to correct[5] the magnitude of $\vec{V}$ in order to reflect the change of orientation of $\vec{V}$ with respect to CT:

$$\varphi_i(x,t) = \frac{\pi}{2} \Rightarrow \cos\varphi_i = 1$$

$$\varphi_{ii}(\mathrm{x},\mathrm{T}) = \alpha \Rightarrow \cos\varphi_{ii} = \cos\left(\sin^{-1}\left(\frac{VT}{CT}\right)\right)$$

$$\Rightarrow \cos\varphi_{ii} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow V = v\sqrt{1 - \frac{v^2}{c^2}}$$

[1] https://en.wikisource.org/wiki/The_Principle_of_Relativity,_and_Non-Newtonian_Mechanics
G. N. Lewis and R. C. Tolman: "The Principle of Relativity, and Non-Newtonian Mechanics", in: “Proceedings of the American Academy of Arts and Sciences”, 1909, 44: 709–726;
The light clock originally was introduced as instructive device for deriving the time dilation formula.

[2] See for example: http://galileo.phys.virginia.edu/classes/252/srelwhat.html

[3] See http://webs.mn.catholic.edu.au/physics/emery/hsc_space_continued.htm and please watch carefully that the vertical distance is given a constant value *L=ct* whereas the velocity vector *vt* is not perpendicular to the time axis *ct*. But the *full* amount of *v* is multiplied by *t*.

[4] http://abyss.uoregon.edu/~js/ast122/lectures/lec20.html

[5] Note: $\cos(sin^{-1}x) = \sqrt{1 - x^2}$